6. UNITS AND LEVELS

6.1 LEVELS AND DECIBELS

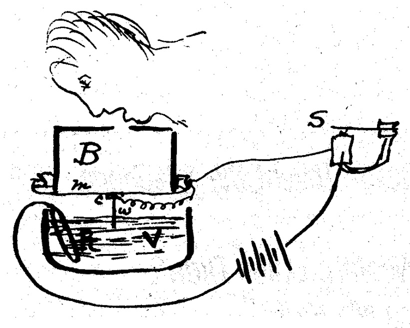


Figure 6.1 Bell's 1876 patent drawing of the telephone

Human response to sound is roughly proportional to the logarithm of sound intensity. A logarithmic level (measured in decibels or dB), in Acoustics, Electrical Engineering, wherever, is always:

$$10\log_{10}\left[\frac{power}{reference\ power}\right] \quad \text{(dB)}$$

An increase in 1 dB is the minimum increment necessary for a noticeably louder sound. The decibel is 1/10 of a Bel, and was named by Bell Labs engineers in honor of Alexander Graham Bell, who in addition to inventing the telephone in 1876, was a speech therapist and elocution teacher.

Sound power level: $L_W = 10 log_{10}\frac{W}{W_{ref}} \qquad W_{ref} = 10^{-12}\ watts$

Sound intensity level:

$$L_I = 10\log_{10}\frac{I}{I_{ref}} \qquad I_{ref} = 10^{-12}\ watts/m^2$$

Sound pressure level (SPL):

$$L_p = 10\log_{10}\frac{P_{rms}^2}{P_{ref}^2} = 20\log_{10}\frac{P_{rms}}{P_{ref}} \qquad P_{ref} = 20\mu Pa = .00002\ N/m^2$$

Some important numbers and unit conversions:

1 Pa = SI unit for pressure = 1 N/m^2 = 10μBar
1 psi = antiquated unit for the metricly challenged = 6894Pa
ρc = characteristic impedance of air = 415 $\frac{kg}{s\cdot m^2}$ = 415 mks rayls (@20°C)
c= speed of sound in air = 343 m/sec (@20°C, 1 atm)

How do dB's relate to reality?

Table 6.1 Sound pressure levels of various sources

Sound Pressure Level (dB re 20 μPa)	Description of sound source	Subjective description
140	moon launch at 100m, artillery fire at gunner's position	intolerable, hazardous
120	ship's engine room, rock concert in front and close to speakers	
100	textile mill, press room with presses running, punch press and wood planers at operator's position	very noise
80	next to busy highway, shouting	noisy
60	department store, restaurant, speech levels	
40	quiet residential neighborhood, ambient level	quiet
20	recording studio, ambient level	very quiet
0	threshold of hearing for normal young people	

6.2 COMBINING DECIBEL LEVELS

Incoherent Sources

Sound at a receiver is often the combination from two or more discrete sources. General case - sources have different frequencies and random phase relation. These are called ***incoherent*** sources. Total energy from two incoherent sources equals the sum of the energy from each. (remember that intensity is proportional to p^2).

Since the total intensity is the sum of the intensity from each individual source, we can calculate the total pressure:

$$P_T^2 = \sum_{i=1}^{n} P_1^2 + \sum P_1^2 + P_2^2 + \dots P_n^2$$

and in dB:

$$10\log_{10}\left(\frac{P_T}{P_{ref}}\right)^2 = 10\log_{10}\sum_{i=1}^{n}\left(\frac{P_i}{P_{ref}}\right)^2 = 10\log_{10}\sum_{i=1}^{n}10^{L_{P_i}/10}$$

Example: What is the combined sound pressure level due to two incoherent sources of 90 and 88 dB respectively? Answer 92.1 dB

Or you can use the graph in Figure 1.6 in our text to combine the levels.

Special Cases to Remember :

- If two incoherent sources have equal levels, the total SPL is 3dB more than each alone.
- A second source which is 10 dB less than the first will add less than .5 dB to the total SPL.

Coherent Sources

If sources are coherent (exactly the same frequency), phase must be considered. The total, combined pressure is:

$$P_T^2 = P_1^2 + P_2^2 + 2P_1P_2\cos(\beta_1 - \beta_2)$$

$$10\log_{10}\left(\frac{P_T}{P_{ref}}\right)^2 = 10\log_{10}\sum_{i=1}^{n}\left(\frac{P_i}{P_{ref}}\right)^2$$

Addition of two coherent sources (totally in phase) adds 6 dB to the level of either alone.

Example Problem: Possible noise sources of an industrial saw include: aerodynamic, mechanical, (motor, blade vibrations). It measures 98 dB @ 1 meter (very loud). In order to determine the contribution of aerodynamic noise, a blade with no teeth is made and measures 91 dB @ 1 meter. How much does the aero noise contribute? Answer: 97dB, therefore the aerodynamic noise is the dominant source

6.3 FUNDAMENTAL RELATIONS

Intensity (far field, no reflections) $$I = \frac{\langle p^2 \rangle}{\rho c} = \frac{P_{rms}^2}{\rho c}$$

Power $$W = \int_S I\,dS$$

Note: In free field, both power and intensity are proportional to p^2

Table 6.2 Variation with distance

Source geometry	**Sound Power**	**Intensity**	**Pressure**
point	independent of r	$1/r^2$	$1/r$
cylinder	independent of r	$1/r$	$1/r^{1/2}$
plane	independent of r	independent of r	independent of r

SPL varies - 3dB/Doubling Distance - cylindrical spreading
- 6dB/Doubling Distance - spherical spreading

6.4 RELATIONS BETWEEN L_P, L_W, AND L_I

If intensity is uniform over area S (assuming spherical spreading)

for a spherical source: $W = I \cdot S$

$$L_W = 10\log_{10}\frac{W}{W_{REF}} = 10\log_{10}\frac{I \cdot S}{W_{REF}}$$

$$= 10\log_{10}\frac{I}{I_{REF}} + 10\log_{10}\frac{S}{S_0}$$

$$S_0 = 1.0 m^2$$

since: $W_{REF} = 10^{-12}\, watts = 10^{-12}\, watts / m^2 x 1.0 m^2 = I_{REF} x 1.0 m^2$

So: $L_W = L_I + 10\log_{10}\frac{S}{S_0}$ $\quad (L_I = L_W\ @\ S = 1.0 m^2)$

and since $W = \frac{P_{RMS}^2}{\rho c} \cdot 4\pi r^2 = I \cdot S$

$$L_W = 10\log_{10}\frac{P_{RMS}^2}{P_{REF}^2} + 10\log_{10}\left[\frac{4\pi r^2 P_{REF}^2}{W_{REF}\,\rho c}\right]$$

using $\rho c = 415 N \sec/m^3$, $W_{REF} = 10^{-12}$ $watts$, $P_{REF} = .00002 Pa$, we get:

$$L_W = L_P + 20\log_{10} r + 11 \ \ dB$$ Equation A

This equation is extremely useful. We can use it to:

1) calculate the SPL at any range if we know the sound power
2) calculate the sound power if we know the SPL at one range

We can also derive a useful equation for relating the sound pressure at any two distances:

$$\Delta SPL = L_{P_1} - L_{P_2} = 20\log_{10}\frac{r_2}{r_1}$$

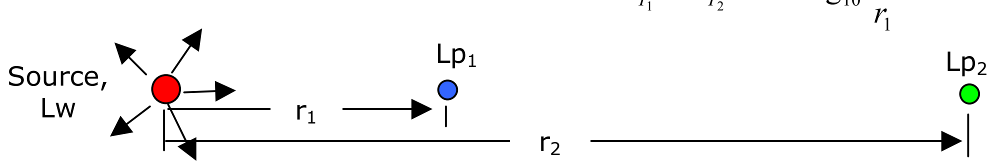


Figure 6.2 Spherical sound propagation

Example problem: How does the SPL change as the distance is doubled for a spherical source?

6.5 SOURCE DIRECTIONALITY

Most sources do not radiate equally in all directions. Example – a circular piston in an infinite baffle (which is a good approximation of a loudspeaker).

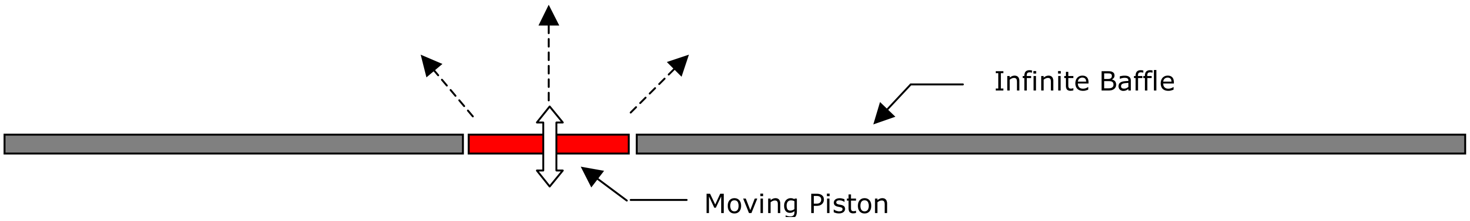


Figure 6.3 A circular piston in a rigid infinite baffle

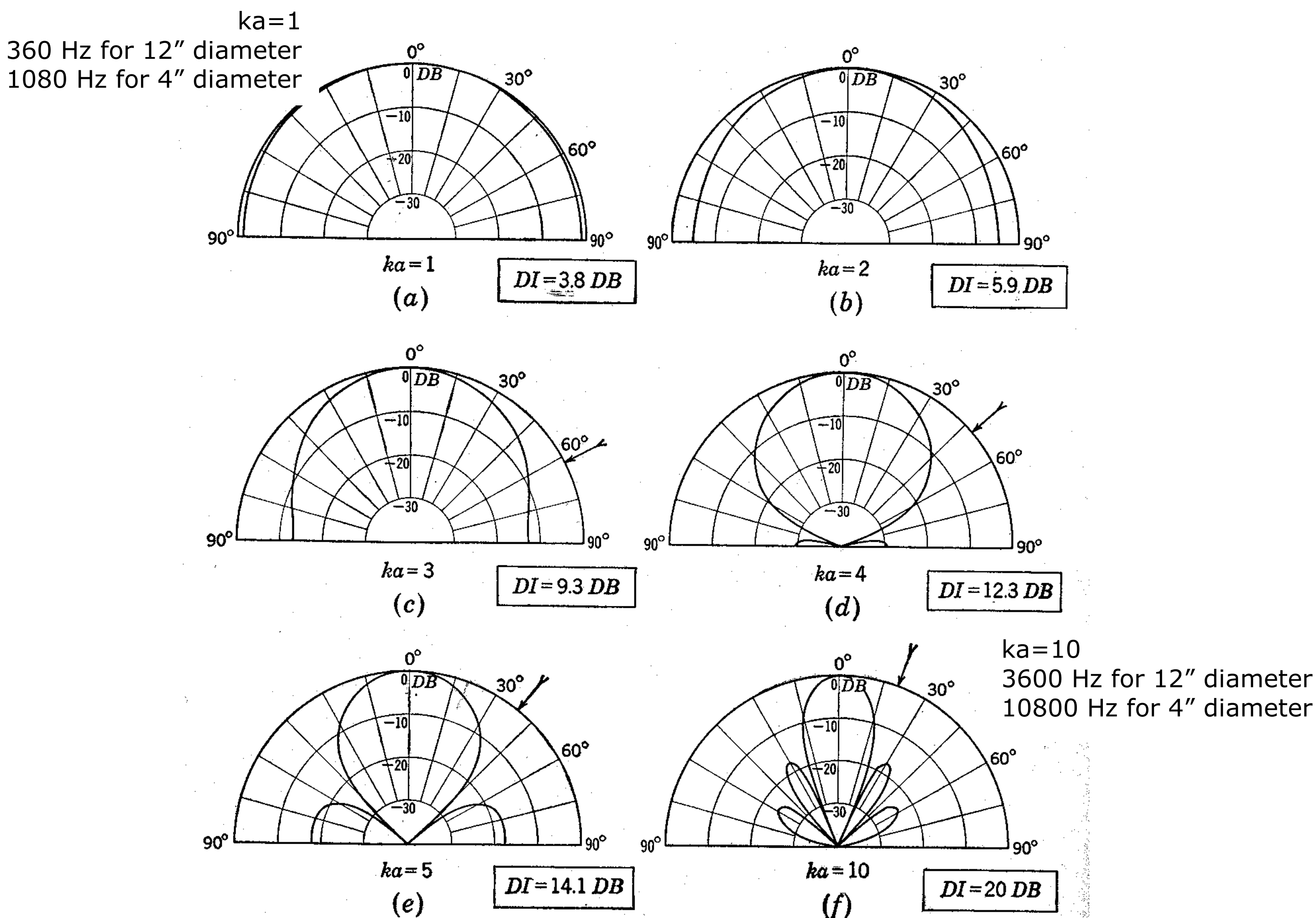


FIG. 4.10. Directivity patterns for a rigid circular piston in an infinite baffle as a function of $ka = 2\pi a/\lambda$, where a is the radius of the piston. The boxes give the directivity index at $\theta = 0°$. One angle of zero directivity index is also indicated. The DI never becomes less than 3 db because the piston radiates only into half-space.

Figure 6.4 Directivity patterns for a circular piston of radius a in an infinite baffle

Define a directivity factor Q (called D_θ in some references) $$Q = \frac{P_\theta^2}{P_S^2} = \frac{I_\theta}{I_{MEAN}}$$

where: P_θ = actual rms sound pressure at angle θ
P_S = rms sound pressure of a uniform point source radiating the same total power W as the actual source

Directivity Index DI: $$DI = 10\log_{10} Q = 10\log_{10} P_\theta^2 - 10\log_{10} P_S^2$$

If DI and W are known, the actual pressure can be calculated by:

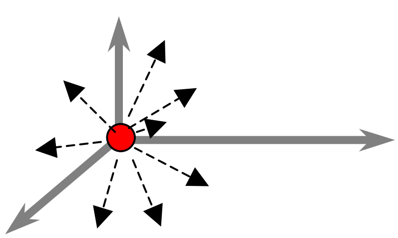
$$P_\theta^2 = Q \cdot P_S^2 = Q\frac{W\rho c}{4\pi r^2}$$

Special Cases:

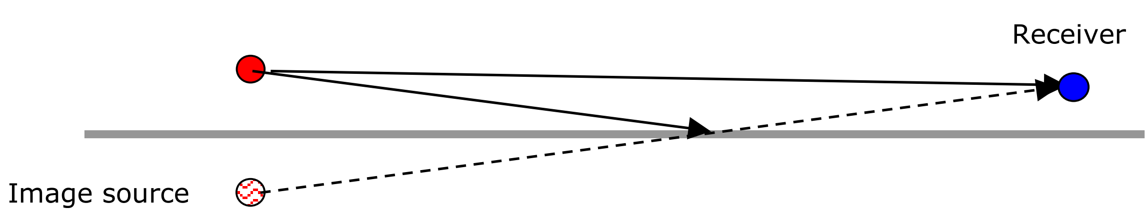
- Hemispherical radiation (point source on a perfectly reflecting surface), DI=3 dB

- Source at the intersection of two planes, DI = 6dB

- Source in a corner (intersection of three planes, DI = 9 dB



- Source above a perfectly reflecting plane



In this case, we have effectively two equal sources with exactly the same phase (the real source and its image). These are coherent sources which will constructively add. The net effect is that the sound pressure will be doubled (assuming that the path length from each source is approximately the same). They can also cancel each other if the path length difference is ½ wavelength. Doubling pressure is equivalent to adding 6 dB to the sound level. Assuming the worst case scenario of perfect constructive addition, equation A becomes:

$$L_P = L_W - 20\log_{10} r - 5 \quad dB$$ for source above a perfectly reflecting plane

Analytical solutions for DI

Analytical solutions for DI are available for some simple sources, such as: piston in infinite baffle, unbaffled piston, cylinders, dipoles. (ref. Acoustics by Beranek)

Example: For a baffled piston or radius *a*, the pressure distribution is:

$$p(r,t) = \frac{\sqrt{2}\, jf\rho_0 u_0 \pi a^2}{r}\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right] e^{j\omega(t-r/c)} \qquad (4.17)$$

where u_0 = rms velocity of the piston

$J_1(\ )$ = Bessel function of the first order for cylindrical coordinates[6]

Now we can update our previous Equation A to include directional effects. For free-field (no reflections, in the far field) propagation from a directional source

$$L_{P_\theta} = L_W - 20\log_{10} r - 11 + DI_\theta$$

For the special case of hemispherical propagation (source located on a perfectly reflecting plane, DI = 3), the apparent power is doubled by the reflection (3 dB increase):

$$L_P = L_W - 20\log_{10} r - 8$$