NEW EXPERIMENTAL TECHNIQUE FOR ENHANCEMENT OF SPATIAL RESOLUTION IN HEAT TRANSFER MEASUREMENTS

Ilyinsky,A.I.+, Rainieri,S., Farina,A. and Pagliarini,G.

Università di Parma,Dipartimento di Ingegneria Industriale
V.le delle Scienze, 43100 Parma
+ Visiting Professor

ABSTRACT

The procedure described in this paper offers a valid mean to handle the problem of random error in temperature measurements planned for the estimation of surface heat flux distribution. The ill-posed inverse heat conduction boundary value problem is solved by using the Fourier Transform Method. This approach enables to use a Digital Signal Processing technique to filter the signal by removing the high frequency components due to noise. The theory has been tested by measuring the temperature of a copper plate where a heat source distribution was provided by some heater elements. The temperature map has been recorded with a thermographic system. Different signal to noise ratios have been considered in order to check the filtering technique applied to the experimental data. The procedure provides accurate results also when a high spatial resolution in the heat source localization is needed.

1. INTRODUCTION

In many engineering applications, like laser technology, diagnostics of heat losses in electronic devices, or industrial process monitoring it is necessary to know the heat flux spatial distribution on the surface of the system under investigation. The surface heat flux distribution is usually estimated by measuring the temperature distribution in the body. This problem is regarded as an inverse heat conduction problem (IHCP). Errors, which are always present to some extent in the temperature measurements, drastically decrease the accuracy in the heat flux restoration. This is because the inverse boundary problem is ill-posed as its solution does not satisfy the requirement of existence, uniqueness and stability under small changes in the measured input temperature values. A quite large number of techniques has been proposed for the solution of this problem. A comprehensive review on the methods of solution of IHCP can be found in Beck et al. (1985) and in Özişik (1993). The standard method follows a least squares approach coupled to a minimization procedure. Both the steady state and the transient heat flux distribution are estimated by minimizing the least squares error between the computed and experimental temperatures. Beck (1963), (1966), (1970), (1977) and (1988) provides examples of the application of the method for both parameter and function estimation. To reduce instability in the solution the introduction of a regularization parameter in the least squares equation was suggested by Tikhonov and Arsenin (1977).

Another approach based on a stochastic description of the experimental errors has been used for example by Fadale et al. (1995). This method, known as maximum likelihood method starts from probabilistic considerations and leads again to a minimization of a functional.

In the present paper a different procedure, intended to overcome the problem of random experimental errors in heat source localization from surface temperature measurements, is suggested. The method is based on the Fourier Transform technique for the solution of partial differential equations and it is here used to find out the heat source distribution in a square domain from the surface temperature map recorded with an infrared camera.

2. FOURIER TRANSFORM METHOD

The Fourier Transform method provides a technique for the solution of self-adjoint positive definite elliptic partial differential equations. This method, coupled to a mode-dependent discretization scheme, can be regarded as a digital signal processing approach, as suggested by Kuo and Levy (1990).

Let's consider the steady state heat conduction equation with energy generation in a 2-D homogeneous domain

$$\lambda \nabla^2 T(x,y) + q(x,y) = 0 \tag{1}$$

defined on the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$ with vanishing temperature along its boundary. By using a standard central-difference discretization scheme the continuous problem satisfied by Eq.(1) is approximated as:

$$\frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{h_x^2} + \frac{T_{i,j-1} - 2T_{i,j} + T_{i,j+1}}{h_y^2} = -\frac{1}{\lambda}\, q_{i,j} \tag{2}$$

where h_x and h_y are the spacings of a grid formed by $(N_x+1)(N_y+1)$ nodes. The replacement of the continuous 2-D Laplace operator by the corresponding discrete form transfers the inverse problem into a class of well-posed problems. This happens because the continuous operator has an unbounded spectrum, while in the discrete domain it has a finite spectral radius, given by $4h_x^{-2} + 4h_y^{-2}$.

The 2-D Laplace operator is self-adjoint and positive definite, therefore its eigenfunctions form a complete basis set with real and positive eigenvalues. It can be easily demonstrated, Marčuk (1984), that on the unitary square domain with zero boundary condition the eigenfunctions are:

$$u_{i,j,k_x,k_y} = 2sin(k_x \pi i h_x) sin(k_y \pi j h_y) \tag{3}$$

and the corresponding eigenvalues:

$$\mathcal{L}_{k_x,k_y} = 4\left[h_x^{-2} sin^2\left(\frac{k_x \pi h_x}{2}\right) + h_y^{-2} sin^2\left(\frac{k_y \pi h_y}{2}\right)\right] \tag{4}$$

where $1 < k_x < N_x - 1$, $1 < k_y < N_y - 1$ and $i, j = 1, 2, ...$

By expanding the solution and the source term in the eigenfunctions:

$$T_{i,j} = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} T_{k_x,k_y} u_{i,j,k_x,k_y} \tag{5}$$

$$q_{i,j} = \sum_{i=0}^{N_x-1} \sum_{j=0}^{N_y-1} q_{k_x,k_y} u_{i,j,k_x,k_y} \tag{6}$$

the finite difference Eq.(2) can be simplified to the diagonal form:

$$-\mathcal{L}_{k_x,k_y} T_{k_x,k_y} + \frac{1}{\lambda}\, q_{k_x,k_y} = 0 \;, \tag{7}$$

where T_{k_x,k_y} and q_{k_x,k_y} are the sine Fourier transforms of $T_{i,j}$ and $q_{i,j}$.

We can easily solve Eq.(7) for q_{k_x,k_y}. Then, by using the inverse sine Fourier transform, the heat flux distribution in the spatial domain is obtained.

If an inhomogeneous condition is to be specified at the boundary the problem can be reverted into an equivalent homogeneous one through the transformation:

$$h^2 q \rightarrow \left(h^2 q - f\right) \;. \tag{8}$$

It is equivalent to consider an effective source term including the inhomogeneous term f. The accuracy in the heat flux restoration is affected by the noise present in the temperature measurement. In particular

$$\|\delta q\| \leq \lambda\, Max\{|\, \mathcal{L}_{k_x,k_y}\, |\} \|\delta T\| \;. \tag{9}$$

On the other hand, the norms of temperature and heat flux are related to each other as:

$$\|T\| \leq \frac{1}{\lambda\, Min\{|\, \mathcal{L}_{k_x,k_y}\, |\}} \|q\| \;. \tag{10}$$

From Eq.(9) and Eq.(10) we obtain the final estimation of the relative accuracy of heat flux restoration:

$$\frac{\|\delta q\|}{\|q\|} \leq \frac{Max\{|\, \mathcal{L}_{k_x,k_y}\, |\}}{Min\{|\, \mathcal{L}_{k_x,k_y}\, |\}} \frac{\|\delta T\|}{\|T\|} \;. \tag{11}$$

From Eq.(4) it can be easily shown that:

$$2\pi^2 \leq \mathcal{L}_{k_x,k_y} \leq \frac{4}{{h_x}^2} + \frac{4}{{h_y}^2} \;. \tag{12}$$

By substituting into Eq.(11) we then obtain

$$\frac{\|\delta q\|}{\|q\|} \leq \frac{1}{\pi^2}(N_x^2 + N_y^2) \frac{\|\delta T\|}{\|T\|} \;. \tag{13}$$

This means that from temperature measurements performed with 1% accuracy on a mesh of 64×64 nodes the heat flux is affected by uncertainties which are one thousand greater.

Hence, the problem of heat flux restoration is well-posed in the discrete domain but it is extremely ill-conditioned.

To overcome this instability it is necessary to increase the signal to noise ratio in the initial input temperature data. This can be easily achieved in the frequency domain since some assumptions on the spectra of both the signal and of the noise can be done. In particular, in almost all practical situations the exact signal has frequency components concentrated in the region of low k_x and k_y values while the spectral components of noise are distributed uniformly over the entire frequency domain.

It follows that for high k_x and k_y values the signal is overcome by noise which thus causes enormous error in the heat flux restoration. Therefore, the Fourier components T_{k_x,k_y} can be suitably filtered by an appropriate 2-D low pass filter with frequency response W_{k_x,k_y}. The filtered signal

$$T^f_{k_x,k_y} = T_{k_x,k_y} W_{k_x,k_y} \tag{14}$$

has a better signal to noise ratio than the rough measured data and can thus be used to recover the heat flux components by using Eq.(7). The filtering technique we have adopted is basically a brick-wall low pass filter convolved with a 2-D Hamming window applied to reduce the effect of rippling in the passband region.

By defining $\Gamma_{xy} = \frac{{k_x}^2}{{F_x}^2} + \frac{{k_y}^2}{{F_y}^2}$, its expression follows as:

$$W_{k_x,k_y}(F_x, F_y) = 0.54 + 0.46\, cos\left[\pi\, (\Gamma_{xy})^{1/2}\right] \tag{15}$$

if $\Gamma_{xy} \leq 1$, or

$$W_{k_x,k_y}(F_x, F_y) = 0 \tag{16}$$

if $\Gamma_{xy} > 1$.

The cut-off frequencies F_x and F_y depend on the magnitude of the noise present in the experimental data and can be determined from a preliminary investigation on the noise equivalent temperature difference of the measuring device. In many situation it can be necessary to apply a non symmetric filter in the frequency domain and this can be easily achieved by selecting different values for F_x and F_y. In this way the filtering procedure can be adjusted according to the measuring system used and to the signal to noise ratio.

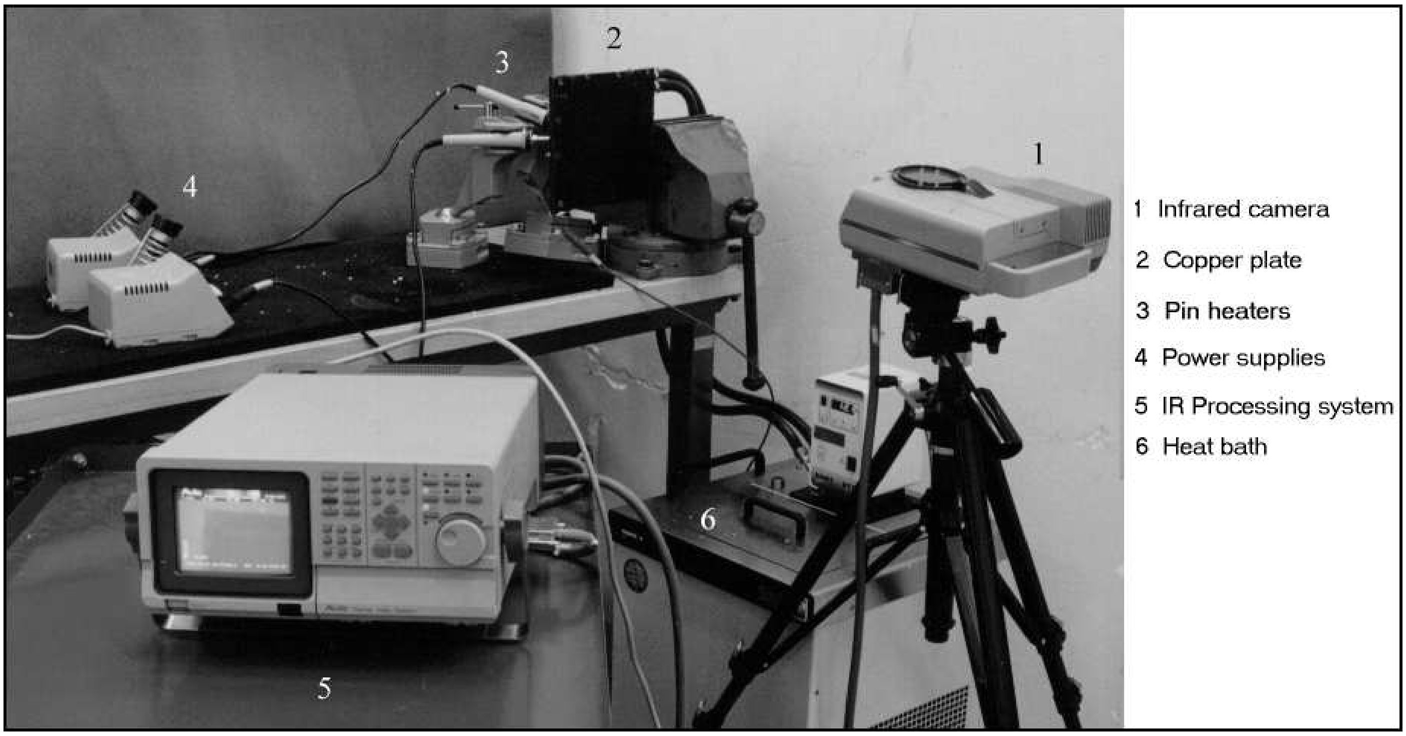

Figure 1: *Experimental setup.*

3. EXPERIMENTAL SETUP

The experimental tests were conducted on a thin square copper plate, 0.85 mm thick and 0.2 m wide. It was fixed on a 3 mm thick copper frame. To control the temperature at the boundary a copper tube with a diameter of 10 mm has been soldered to the frame and connected to a heat bath with a refrigerant unit. Figure 1 shows the experimental setup.

To produce different heat source distributions two pin heaters supported by a clamp were put in direct contact with the plate. The dimension of the pin was about 3 mm, and the power output was adjustable within a maximum value of 50 W for each of them.

The temperature distribution on the surface was measured by a thermographic system (AVIO Thermal Video System TVS 2000ST by Nippon Avionics CO). The temperature map was recorded in a color image with a resolution of 256 horizontal $\times$ 200 vertical pixels with 8-bit level. The noise equivalent temperature difference of the system is $0.2K$. This value can be reduced by averaging over more images. In particular the system enables to average over 2^n images, with n varying from 1 to 8.

The surface of the slab was painted with a black opaque painting and its emissivity has been estimated by measuring the temperature in some points with type-T thermocouples. The value $\epsilon \approx 0.96$ was found.

4. RESULTS

To test the noise level of the whole system a preliminary measure was performed while maintaining the plate in thermal equilibrium with the ambient. The data were recorded by averaging over different number of images. Figure 2 shows the temperature distribution in the frequency domain recorded with no average and by averaging over $4, 16$ and 256 images.

The spectra have been obtained via a 32×32 points sine Fourier transform. Figure 3 shows the mean value and the standard deviation of the normalized temperature data versus the number of images.

Although increasing the averaging the effect of the noise is greatly reduced, some residual frequency components are still present also in the case of the average over 256 images.

The residual frequency components are probably due to the mechanical scanning device of the infrared camera. However the information we have about the measuring system do not allow to clearly identify their origin. In the following analysis this effect is taken into account by filtering from the signal the specific residual frequency components found with this preliminary analysis of the infrared camera response.

Other effects like radiation losses or other modulation effects due to the sensor itself which produce a degradation of the image, without introducing other components in the frequency domain, have been neglected. The procedure here implemented focuses in fact attention on the problem of random experimental error in inverse steady state heat conduction problem and, in particular, on enhancing the spatial resolution in heat source localisation.

In figure 4 the normalized measured temperature is compared with the noise level of the system estimated in isothermal condition. In particular the temperature distribution in the frequency domain along the k_x and k_y directions is compared to the noise components, averaged along the same directions, shown in figure 2 for $M = 1$ and $M = 256$.

From the data it can be deduced that increasing k_x and k_y the effect of the noise becomes dominant over the signal. Especially in the k_x direction the noise hides the signal even in the case of 256 images already for k_x greater

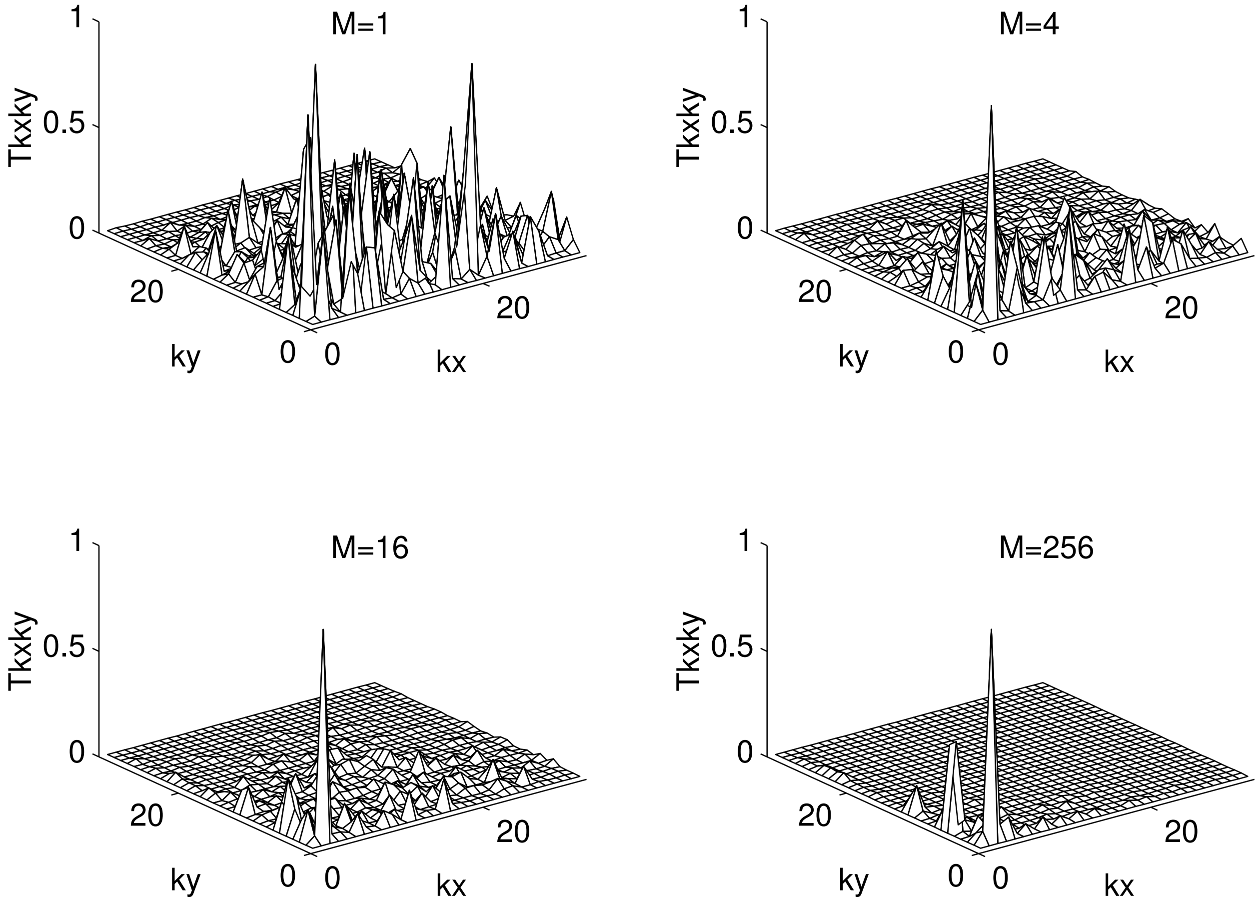

Figure 2: *Normalized temperature distribution in the frequency domain obtained by averaging over different numbers of images.*

than 10. From this comparison the cutoff frequencies for the low-pass filter, F_x and F_y in Eq.(15) and Eq.(16), can be estimated. In the present application they were selected in the range $10 < F < 18$, for both F_x and F_y.

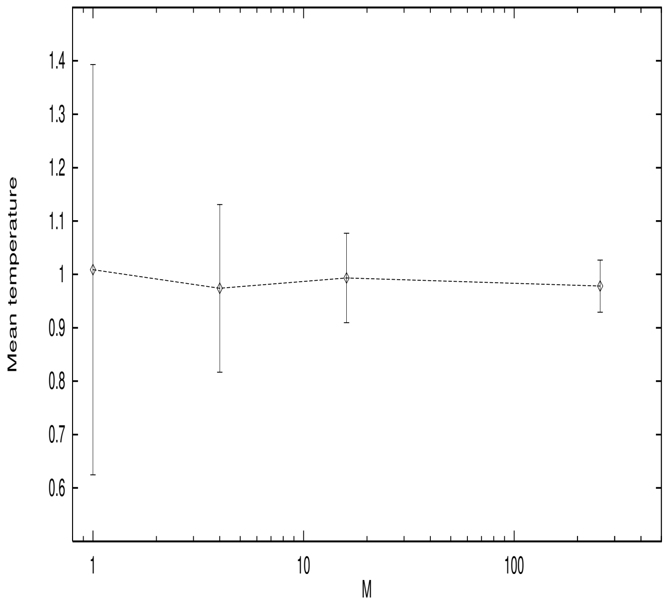

Figure 3: *Mean value and standard deviation of normalized temperature data versus the number of images.*

The heat flux distribution was created by placing two pin heaters in contact with the copper slab at different distances. Distances between the pin axis of $23, 13, 8, 6, 5$ mm were tested. The distance to pixel number ratio has been estimated to be 0.78 mm/pixel in the horizontal direction and 0.69 mm/pixel in the vertical direction. The two peaks in the restored heat flux distribution appear to be resolved up to distances of 5 mm on the tested copper plate. Figures 5 and 6 show the temperature and the recovered heat sources distribution in a 32×32 pixels grid corresponding to a distance of 5 mm between the pin heaters.

It is worth saying that from the rough temperature distribution it is not possible to distinguish the two peaks and that without the filtering procedure the data do not allow to recover the effective surface heat flux distribution. Figure 7 clearly shows that it is impossible to estimate the heat source position without rejecting the high frequency components due to the experimental noise.

Measurements were performed also with three heat sources having different intensities. Figure 8 shows that the heat flux distribution is successfully recovered in this case, too.

The real and the estimated distances between the two heat sources considered in the experiment are compared in figure 9.

The solution method for the restoration of heat flux

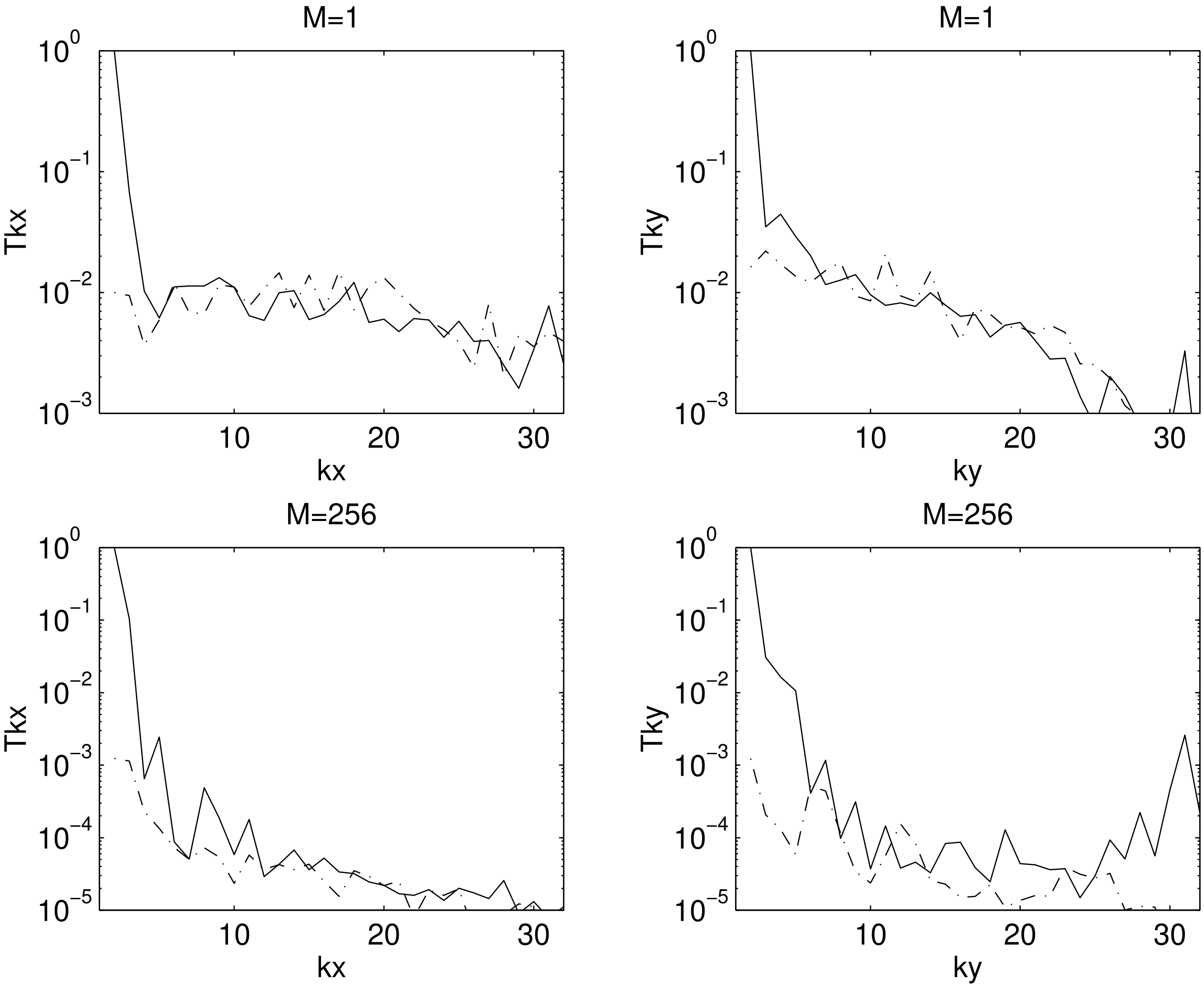

Figure 4: *Normalized measured temperature (solid line) and pure noise (dashed line) versus* k_x *and ky directions with no average and averaged over* 256 *images.*

distribution here presented requires basically one direct and one indirect transformation from the space domain to the frequency domain. From the computational point of view, if a fast Fourier transform algorithm is used, the complexity is $O(N^2 ln(N))$. In the standard method based on the least squares approach the minimization procedure leads instead to the solution of a system of linear equations. The computational complexity is in this case $O(N^3)$ if, for instance, a Gaussian-elimination solution is used. The disadvantage of the Fourier transform method is that it can be easily applied only to square or rectangular domain. The matrix approach has instead a more general applicability and can be used also where an irregular domain or a nonuniform grid are considered.

However it can be noted that, if infrared thermography is used as measuring technique, the temperature map can always be converted to a rectangular or square image with equally spaced pixels in each direction. The other traditional measuring devices, like thermocouples, are in any case not suitable for this application because a high spatial resolution is needed to discretize the domain and to make a good sampling of the signal.

5. CONCLUSIONS

A new technique for heat source identification based on the expansion of the temperature distribution in eigenfunctions of the discrete 2-D Laplace operator has been developed. To treat the ill-posed problem of heat flux restoration a filtration technique typical of signal processing analysis has been used. The theory has been tested on experimental data obtained from a series of thermographic temperature measurements taken on a locally heated copper square plate. The heat source distribution in the system has been successfully restored and the method has then found a validation in the experimental results. The processing of temperature data enables to drastically increase the spatial resolution in surface heat flux restoration. The developed technique allows to reduce the effect of noise in temperature data and can thus be applied in a variety of engineering applications.

6. ACKNOWLEDGMENTS

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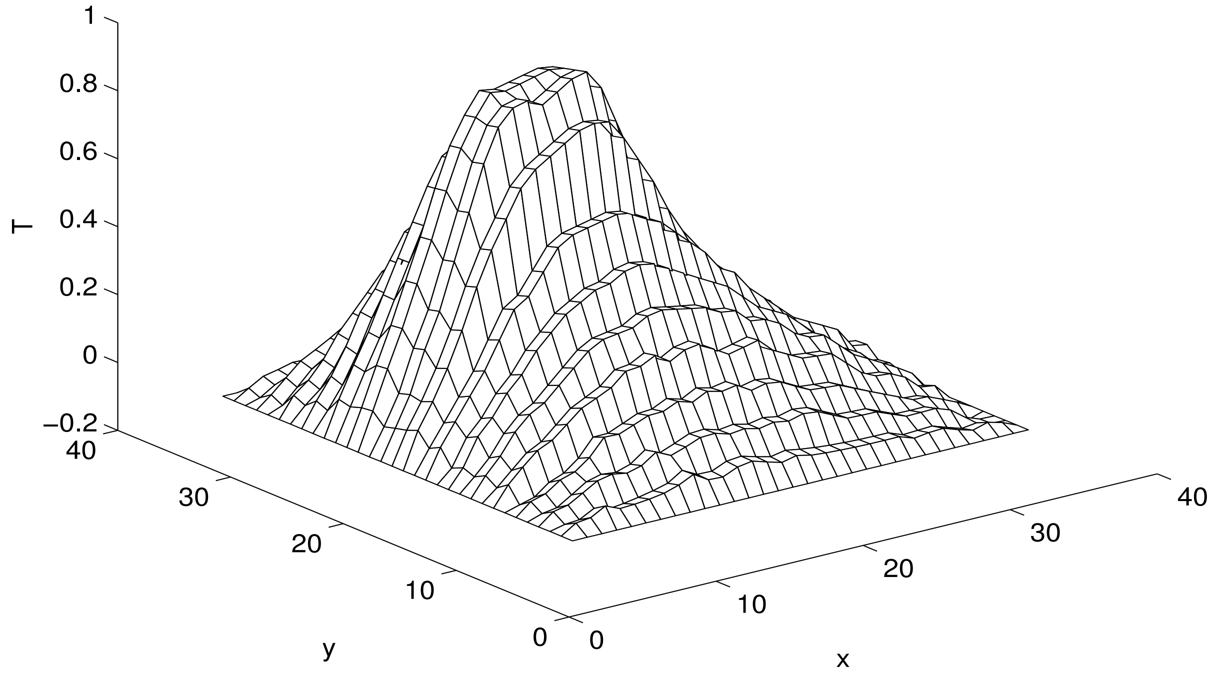

Figure 5: *Normalized temperature distribution.*

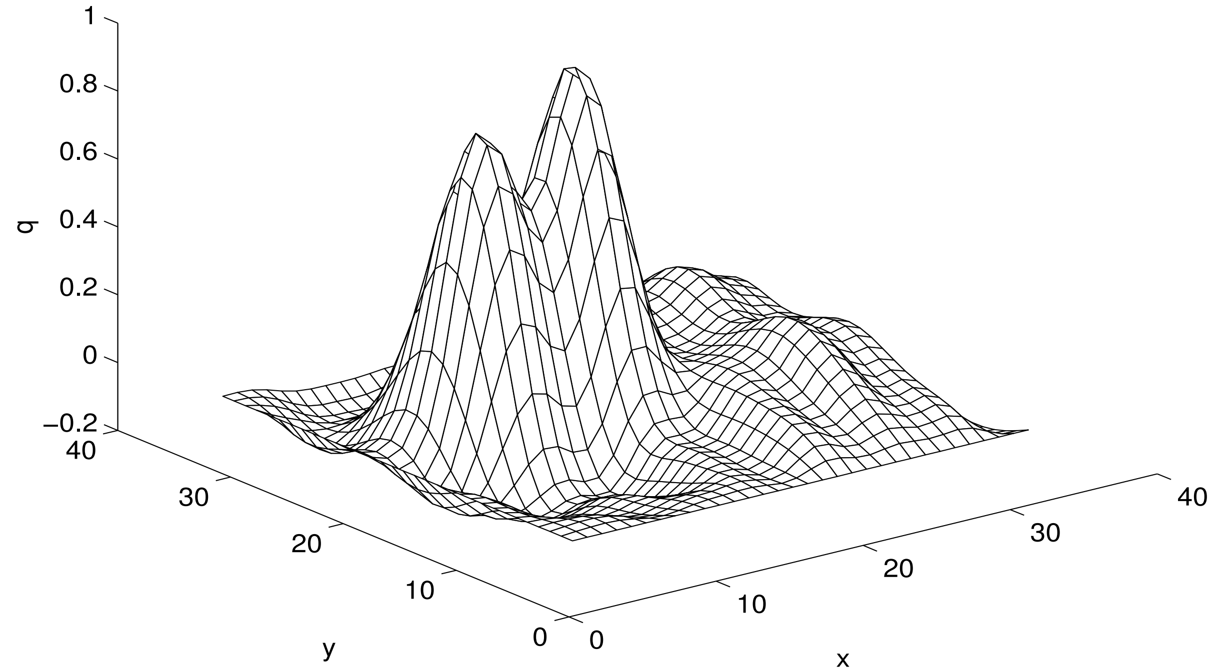

Figure 6: *Normalized heat flux distribution with filtration.*

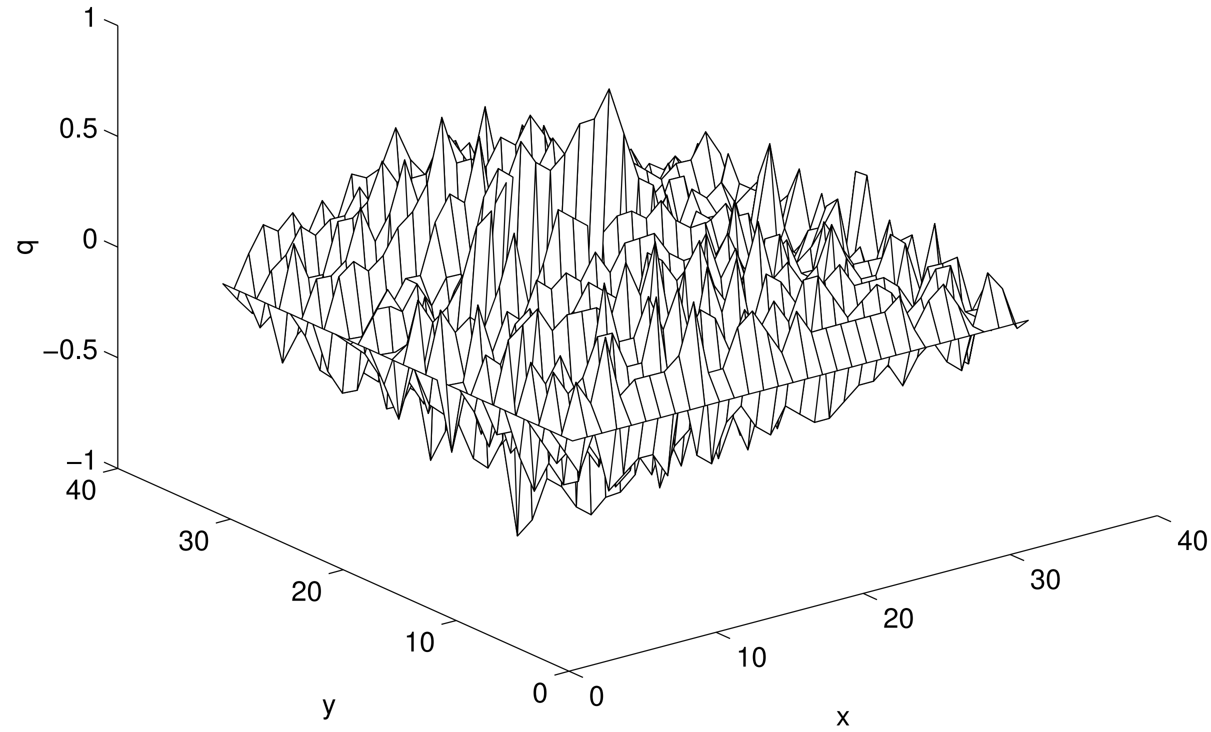

Figure 7: *Normalized heat flux distribution without filtration.*

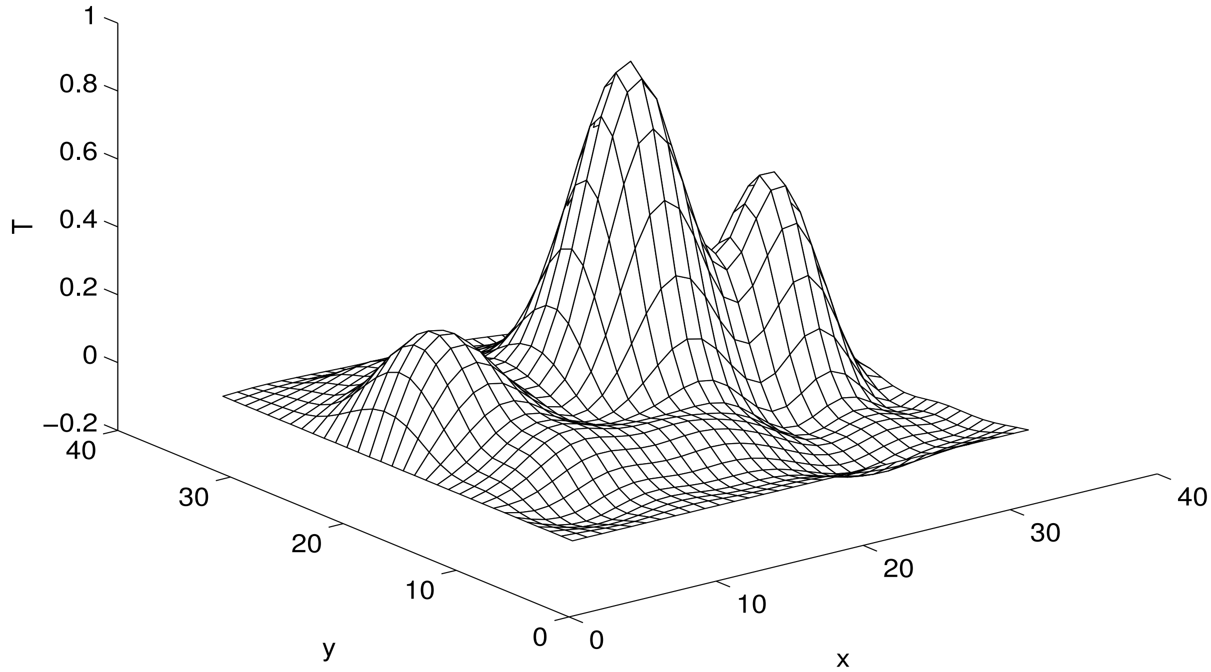

Figure 8: *Normalized heat flux distribution with filtration.*

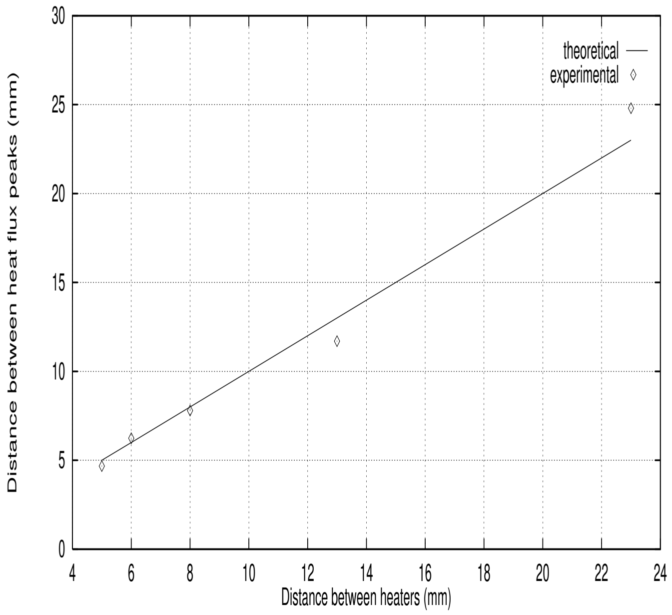

Figure 9: *Recovered distances between heaters.*

NOMENCLATURE

f	inhomogeneous term
F_x, F_y	cutoff frequencies $(1/m)$
$h_x = \dfrac{1}{N_x}, h_y = \dfrac{1}{N_y}$	grid spacings
k_x, k_y	wave numbers $(1/m)$
M	number of images
N_x, N_y	number of grid points
q	heat generation (W/m^3)
T	temperature (K)
u_{i,j,k_x,k_y}	eigenfunctions of the 2D Laplace operator
$W_{k_x,k_y}(F_x, F_y)$	filter matrix
λ	thermal conductivity $(W/m \cdot K)$
ϵ	emissivity
Γ_{xy}	$\dfrac{{k_x}^2}{{F_x}^2} + \dfrac{{k_y}^2}{{F_y}^2}$
$\mathcal{L}$	eigenvalues of the 2D Laplace operator

Superscripts

f	filtered

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